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Statistical features of transition to stable film boiling

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Abstract

The development of transition boiling modes is virtually the formation and spreading process of dry patches induced by applied superheat or heat fluxes. Given that the interactions among multiple dry patches with different sizes, transition boiling can only be described in statistical way. In this paper, dry patches statistical distribution function was obtained and the stochastic feature of transition to stable film boiling was then discussed. The new academic implications on classical MHF (minimal heat flux) model have been revealed. The present investigations tried to make a renewed effort to the understanding of transition boiling. Available experimental results revealed that the present analysis was more reasonable than traditional one in framework.

Keywords: Boiling; Transition boiling; Statistical analysis; Dry patch

1. Introduction

Transition boiling bridges the nucleate boiling and film boiling, which is encountered in a number of applications, including metallurgical quenching processes, and immersion cooling of high temperature components. Although having been studied for decades, transition boiling still holds its place as the least understood of the several boiling mechanisms [1]. Such an occurrence is attributed to the complexity of mechanisms controlling the transition boiling heat transfer, and the difficulty of performing experiments. Since the pioneering work of Berenson [2], researchers noted that the transition boiling process incorporated liquid–solid–vapor triple phases interactions. The heat transfer mechanism was hence regarded as a combined process including transient heat conduction, nucleation boiling, and film boiling heat transfer modes [3]. It is no doubt that dry patches play important roles during whole transition boiling processes, for the dry patches would form at the high-density level and at the great size in transition boiling [4]. The wetting ratio dramatically falls as wall superheat increases. Till stable film boiling has established on the heated surface, the wetting ratio approaches zero [5].

It is not yet unanimous regarding the dynamic features in transition boiling. Witte and Lienhard [6] and Ramilison and Lienhard [7] reported the occurrence of sudden "jumps" between nucleate boiling and film boiling regimes. Large dry patches once emerge, would flush and cover the entire heated surface. However, Bui and Dhir [8] tried, but failed to destabilize the transition-boiling mode by wiping the surface with a brush. Dhuga and Winterton [9] had neither observed any jumps in their transition boiling experiments. We claimed that, although certainly possible, jumps in transition boiling curve were not inevitable [10]. Restated, the occurrence of jumps in transition boiling curve is conditional and statistical. There exists no comprehensive framework to interpret these experimental findings until now [11].

Actually, available literatures on transition boiling mostly encounter a time/surface-averaged procedure in which a stationary boiling process is assumed. While during transition boiling, the wall temperature is extremely non-uniform for the stochastic formation/rewet of dry patches. For the inter-

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Nomenclature

C_p	specific heat at constant pressure
F	fluctuation growth rate
h_{fg}	latent heat
k	drift growth rate
N	dry patch number distribution function
p	pressure
P	dry patch probability function
Pr	Prandtl number
q	dry patch production rate
r	dry patch radius
R	safe probability function
R_a	roughness
s^*	critical radius
t	time
T	temperature

Greek symbols

ξ	dimensionless number
ρ	density
σ	surface tension
λ	thermal conductivity
$\hat{a}$	constants
δ	Dirac delta function
θ	defined parameter
γ	defined parameter

Subscripts

l	liquid
MHF	minimal heat flux
v	vapor
w	wall

actions among multiple dry patches with different sizes, it is becoming apparent that transition boiling can only be described in statistical way. Dry patch dynamic feature could be largely masked by the averaging [12].

From above survey, it may be concluded that a new perspective on transition boiling heat transfer is desired [13]. In this paper, from the view of nonlinear interaction mechanism and inclusion of nonuniform dry patches size distribution, we propose stochastic analysis on dry patches growth in transition boiling system. Dry patches statistical distribution function is obtained and the stochastic feature of transition to stable film boiling is then discussed. The research reported herein is directed in attempt to describe stochastic features in transition boiling process in a new fashion. The new academic implications on classical MHF (minimal heat flux) model have been revealed. Comparisons to our previous experimental results show that present theoretical analysis is not only beneficial to the current understanding of the transition boiling mechanisms, but also potential for the prediction of the transition boiling heat transfer with more accuracy.

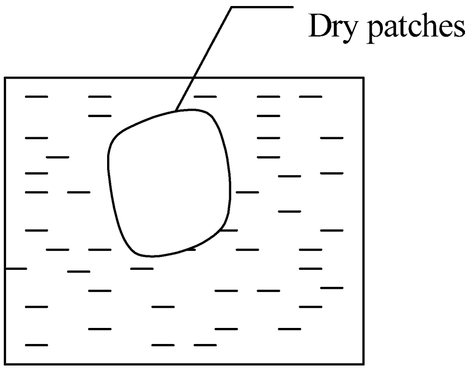

Fig. 1. A single dry patch is surrounded by infinite liquid phase.

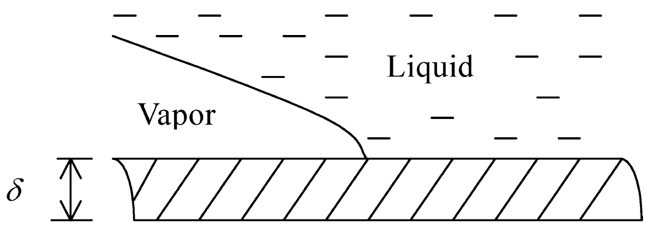

Fig. 2. Analytical model.

2. Physical descriptions

2.1. The case for the existence of single dry patch

If we mainly considered a single dry patch surrounded by an infinite liquid phase, as shown in Fig. 1, the case could be simplified as an illustrative model with lateral heat conduction, as shown in Fig. 2. Physically, when dry patch grows up to a certain size, it cannot be rewetted any longer. This size was often represented as critical radius of dry patch. Typical research could be found in references [14,15]. Let us assume the critical radius of dry patch obtained with method in references [14,15] is s^*. Obviously, as shown in Fig. 2, if the single dry patch is larger than s^* in radius, the vapor front will surely propagate toward the liquid phase till the occurrence of stable film boiling. Propagation is deterministic, not uncertainly [14]. Doubtless, this kind of analyses can only approximately reflects the feature of transition to stable film boiling, for a lot of dry patches with all kinds of sizes often simultaneously exist in real transition boiling.

2.2. The case for the existence of multiple dry patches

Transition boiling system generates highly complicated phenomena. Transition boiling is a dynamic phenomenon with vivid dry patches generation, growth and being rewetted under the action of superheat or heat fluxes. The whole

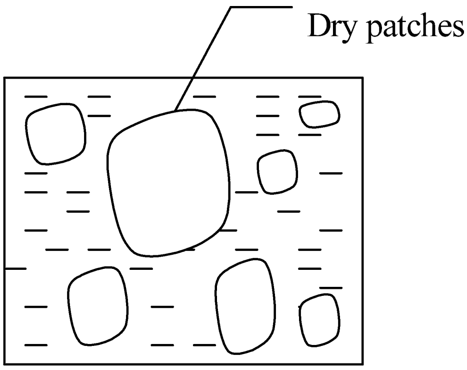

Fig. 3. The co-existence of multiple dry patches.

transition boiling may be principally imagined as the formation, growth and spreading of dry patches, as roughly shown in Fig. 3. The transition boiling process is a typical non-equilibrium irreversible kinetic process. According to well-known many-bodies theory in quantum mechanics and statistics mechanics, a complex system with multiple elements interactions can only be described by stochastic theory and statistical way [16]. In this connection, the law of transition boiling evolution process with multiple dry patches interactions is stochastic, but not deterministic. Hence the basic law of the whole process is also statistical.

From the stochastic features of the occurrence of dry patches, a logical conclusion can be drawn that the occurrence of critical dry patch radius also shows stochastic behaviors and is related to certain probability value from viewpoints of stochastic theory. Critical dry patch (with radius equal to or larger than s^*) derived by deterministic method (the case for the existence of single dry patch) cannot necessarily spread. We can only say that this critical dry patch has a certain spreading probability. Detailed analysis can be found in part 4 of this paper.

3. Dry patch distribution model

Now we give the differential equation describing the stochastic process. Let t denote the time that a wall is subjected to applied heat fluxes, r denote dry patches radius, and $\dot{r}$ denote dry patches growth rate. Because the dry patch radius can be regarded as the average background superimposed by inhomogeneous fluctuation, the dry patch growth rate must obey the following generalized Langevin equation [16]

$$\dot{r} = k(r) + F(t) \tag{1}$$

Where $k(r)$ is drift growth rate, which is determined by outer conditions. $F(t)$ is fluctuation growth rate, which is determined by inhomogeneous fluctuation. By adjusting terms $k(r)$ and $F(t)$, Eq. (1) can describe dry patch development at any regimes. In general, dry patch growth process may be approximately regarded as a Markov process and the fluctuation $F(t)$ may be assumed to have a Gaussian distribution for the convenience of calculation, i.e.,

$$\langle F(t)F(t')\rangle = Q\delta(t-t') \tag{2}$$

Here δ is Dirac function [16] and Q is the fluctuation growth coefficient. Sometime fluctuation growth coefficient varies with time and should be written in $Q(t)$. But similar analysis can be conducted.

Eqs. (1) and (2) are Langevin-typed descriptions for stochastic processes. In term of stochastic theory, the generalized Fokker–Planck equation [16], which corresponds to the generalized Langevin Eqs. (1) and (2), is as follows

$$\frac{\partial P(r,t)}{\partial t} = -\frac{\partial}{\partial r}\big[k(r)P\big] + \frac{Q}{2}\frac{\partial^2 P}{\partial r^2} \tag{3}$$

This is the differential equation describing the stochastic growth process of the dry patch, where $P(r,t)$ is the probability of dry patches between r and $r + \mathrm{d}r$ at time t. Obviously $P(r,t)$ satisfies the normalization condition

$$\int_0^\infty P(r,t)\,\mathrm{d}r = 1 \tag{4}$$

In general, in a real boiling system, a large number of dry patches form and grow simultaneously under action of applied heat fluxes. It is necessary to derive a differential equation describing the evolution process of a number of dry patches.

Superheat is a main factor for transition boiling. Let us introduce a dimensionless parameter $\xi = \frac{\rho_v h_{\mathrm{fg}}}{\rho_l C_{pl}\Delta T}$, which reflects the effect of superheat $\Delta T = T_w - T_l$. Under the action of heat flux, the average dry patches number for radius r at ξ is $N(r,\xi)\,\mathrm{d}r$, the average dry patches number for radius between r and $r + \mathrm{d}r$ at $\xi - \mathrm{d}\xi$, is $N(r, \xi = \mathrm{d}\xi)\,\mathrm{d}r = (N - \frac{\partial N}{\partial \xi}\,\mathrm{d}\xi)\,\mathrm{d}r$ and net increase in dry patch number at ξ to $\xi - \mathrm{d}\xi$ is

$$-\frac{\partial N}{\partial \xi}\,\mathrm{d}\xi\,\mathrm{d}r \tag{5}$$

The increase of dry patches number comes from two parts: one is due to dry patch growth, and the other is due to formation. The averaged dry patches number coming from r into $r + \mathrm{d}r$ due to growth at ξ to $\xi - \mathrm{d}\xi$ is $(k(r)N)_r$ ($k(r)$ is drift dry patch growth rate). The averaged dry patch number coming out of $r + \mathrm{d}r$ from initial radius between r and $r + \mathrm{d}r$ for growth is

$$\big[k(r)N\big]_{r+\mathrm{d}r}\,\mathrm{d}\xi = \left[\big(k(r)N\big)_r + \frac{\partial}{\partial r}\big(k(r)N\big)_r \mathrm{d}r\right]\mathrm{d}\xi \tag{6}$$

So the net increase of the averaged dry patches number between r and $r + \mathrm{d}r$ for growth is

$$-\frac{\partial}{\partial r}\big(k(r)N\big)\,\mathrm{d}r\,\mathrm{d}\xi \tag{7}$$

The new averaged dry patch number between r and $r + \mathrm{d}r$ at ξ to $\xi - \mathrm{d}\xi$ is

$$q(r,\xi)\,\mathrm{d}r\,\mathrm{d}\xi \tag{8}$$

$q(r,\xi)\,\mathrm{d}r$ is averaged dry patches number in unit between r and $r+\mathrm{d}r$ at ξ for formation of new dry patches.

Combining Eqs. (5), (7) and (8) yields

$$-\frac{\partial N(r,\xi)}{\partial \xi} = q(r,\xi) - \frac{\partial}{\partial r}\big[k(r,\xi)N(r,\xi)\big] \tag{9}$$

For the convenience of calculation, $q(r,\xi)$ can be reasonably supposed to be the form

$$q(r,\xi) = q(\xi)\delta(r) \tag{10}$$

δ is Dirac function, Eq. (9) is then changed into

$$-\frac{\partial N}{\partial \xi} = q(\xi)\delta(r) - \frac{\partial}{\partial r}(kN) \tag{11}$$

Where the first term is the increase for new production of dry patches, the second term is the increase for growth. The above equation satisfies the initial condition and boundary condition

$$N(r,\xi\to\infty) = 0 \tag{12a}$$

$$N(r\to\infty,\xi) = 0 \tag{12b}$$

In term of Eqs. (11) and (12), $N(r,\xi)$ can be obtained for known $q(\xi)$ and $k(r,\xi)$. According to $P(r,\xi)\,\mathrm{d}r = N(r,\xi)\,\mathrm{d}r/N(\xi)$, we can obtain $P(r,\xi)$, which is probability that we can find the dry patches between r and $r+\mathrm{d}r$ among all sizes. $N(\xi) = \int_0^\infty N(r,\xi)\,\mathrm{d}r$ is the total dry patch density.

It is useful to get the solutions of evolution differential equations. In term of basic theory of differential equation, Eqs. (9)–(12) can be combined as

$$\begin{aligned}
&-\frac{\partial N(\xi,r)}{\partial \xi} + \frac{\partial}{\partial r}\big[\overline{k(r,\xi)}N(\xi,r)\big] = 0\\
&N(\xi\to\infty,r) = 0\\
&n(\xi,r\to\infty) = 0\\
&\big[k(r)N\big]_{r=0} = q(\xi)
\end{aligned} \tag{13}$$

$\overline{k(r,\xi)}$ is averaged growth rate. $q(\xi)$ may approximately be equal to nucleation rate and can be found in literature [17]

$$q(\xi) = 218.8(Pr)^{1.03}\left(\frac{1}{\gamma}\right)\theta^{-0.4}\left(\frac{\rho_v h_{\mathrm{fg}}}{\rho_l C_{pl}}\right)^3 \xi^{-3} \tag{14}$$

where

$$\theta = A + B\left(\frac{Ra\,P}{\sigma}\right) + C\left(\frac{Ra\,P}{\sigma}\right)^2$$

$$y = \sqrt{\frac{\sigma_w}{a_l}\frac{\rho_w C_{pw}}{\rho_l C_{pl}}}$$

(Subscripts w is for wall, l is for liquid, and p is for pressure, then C_{pw} is specific heat for wall, et al.)

Considering dry patches as large coalescence bubbles, its averaged growth rate is approximately [18]

$$\overline{k(r,\xi)} = \hat{a}\frac{\lambda_l}{\rho_l C_{pl}}\xi^{-1} \tag{15}$$

Eq. (13) then yields the solution

$$N(\xi,r) = \frac{218.8(Pr)^{1.03}(1/\gamma)\theta^{-0.4}(\frac{\rho_v h_{\mathrm{fg}}}{\rho_l C_{pl}})^3 C_{pl}\rho_l}{\hat{a}\lambda_l\xi^2} \times \exp\left(-\frac{2\rho_l C_{pl}}{\hat{a}\lambda_l}r\right) \tag{16}$$

Eq. (16) physically means that dry patches density squarely increases with decreasing ξ and exponentially decreases with increasing dry patch radius.

Dry patch probability can be obtained by $P(r,\xi)\,\mathrm{d}r = N(r,\xi)\,\mathrm{d}r/N(\xi)$, and it yields

$$P(\xi,r) = \frac{2\rho_l C_{pl}}{\hat{a}\lambda_l}\exp\left(-\frac{2\rho_l C_{pl}}{\hat{a}\lambda_l}r\right) \tag{17}$$

The result of Eq. (17) means that dry patch probability exponentially decreases with increasing dry patch radius.

Obviously dry patch probability satisfies normalization condition

$$\int_0^\infty P(r,\xi)\,\mathrm{d}r = 1 \tag{18}$$

4. Statistical feature on transition to stable film boiling

Transition to stable film boiling is important for a better understanding of a number of thermal processes in industrial research facilities and thus attracts a lot of research. However, the basic phenomena are still elusive today. Basing on physical vision of part 2 and results of part 3 of this paper, here we further give statistical analyses from stochastic viewpoints. Stochastic features of dry patches mean that the occurrence of critical dry patch radius also shows stochastic behaviors and is related to certain probability value. Assuming the critical radius of dry patch obtained with method in references [14,15] is s^*, the following equation can be obtained by substituting it into Eq. (17)

$$P(\xi,s^*) = \frac{2\rho_l C_{pl}}{\hat{a}\lambda_l}\exp\left(-\frac{2\rho_l C_{pl}}{\hat{a}\lambda_l}s^*\right) \tag{19}$$

$P(\xi,s^*)$ is the probability function for the occurrence of critical dry patches, which physically means that the occurrence of dry patch with critical radius shows stochastic behaviors and is related to certain probability value.

Apparently, probability distribution of critical dry patches satisfies the normalization condition

$$\int_0^\infty P(\xi,s^*)\,\mathrm{d}s = 1 \tag{20}$$

The total number of dry patches $N(\xi)$ in unit area is

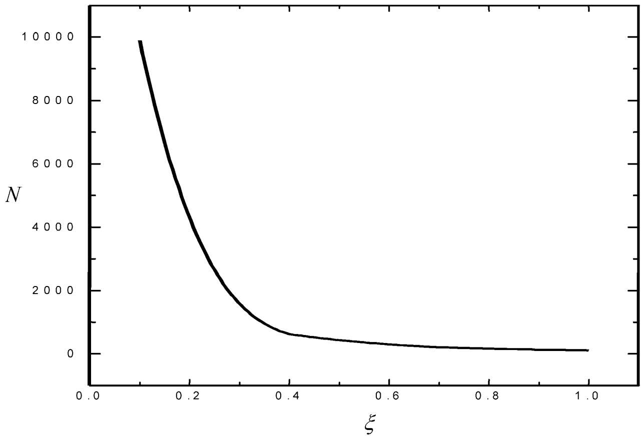

Fig. 4. Dry patch number density as a function of ξ.

$$N(\xi)=\int_0^\infty N(\xi,r)\,\mathrm{d}r$$
$$=\int_0^\infty \frac{218.8(Pr)^{1.03}(\frac{1}{\gamma})\theta^{-0.4}(\frac{\rho_v h_{\mathrm{fg}}}{\rho_l C_{pl}})^3 C_{pl}\rho_l}{\hat{a}\lambda_l\xi^2}\times\exp\left(-\frac{2\rho_l C_{pl}}{\hat{a}\lambda_l}r\right)\mathrm{d}r \tag{21}$$
$$=\frac{218.8(Pr)^{1.03}(\frac{1}{\gamma})\theta^{-0.4}(\frac{\rho_v h_{\mathrm{fg}}}{\rho_l C_{pl}})^3 C_{pl}\rho_l}{2\xi^2}$$

Choosing typical thermo-physical properties, the illustrative result of Eq. (21) is shown in Fig. 4. Obviously, the dry patch number squarely decreases with increasing ξ.

The probability for the occurrence of transition to stable film boiling, P_{MHF}, is equal to the probability satisfying the condition that any single one dry patch radius being larger than s^*, while others still being less than s^*, therefore, we have

$$P_{\mathrm{MHF}}=1-\left[1-\int_{s^*}^\infty P(\xi,r)\,\mathrm{d}r\right]^{N(\xi)} \tag{22}$$

Physically, the probability is unit when the size of critical dry patch is zero. While the probability will be zero when the size of critical dry patch is infinite. Generally the probability for the occurrence of transition to stable film boiling ranges between 0 and 1. For $\int_{s^*}^\infty P(\xi,r)\,\mathrm{d}r \ll 1$, and $N(\xi)\gg 1$, Eq. (22) can be simplified as

$$P_{\mathrm{MHF}}=1-\exp\left[-N(\xi)\int_{s^*}^\infty P(\xi,r)\,\mathrm{d}r\right]$$
$$=1-\exp\left[-N(\xi)\exp\left(-\frac{2\rho C_p}{\hat{a}\lambda}s^*\right)\right] \tag{23}$$

The result of Eq. (23) is shown in Fig. 5.

The occurrence of transition to stable film boiling often means danger incipience of industrial heat exchanger. P_{MHF} can also be defined as danger exponent that physically means danger may occur in boiling systems. In a more explicit way, we define safety probability function, which physically means the probability that the boiling system keeps below stable film boiling, as

$$R=1-P_{\mathrm{MHF}}=\exp\left[-N(\xi)\exp\left(-\frac{2\rho C_p}{\hat{a}\lambda}s^*\right)\right] \tag{24}$$

The result of Eq. (24) is shown in Fig. 6. It is our hope that the same discussion may be applied to the occurrence of critical heat flux.

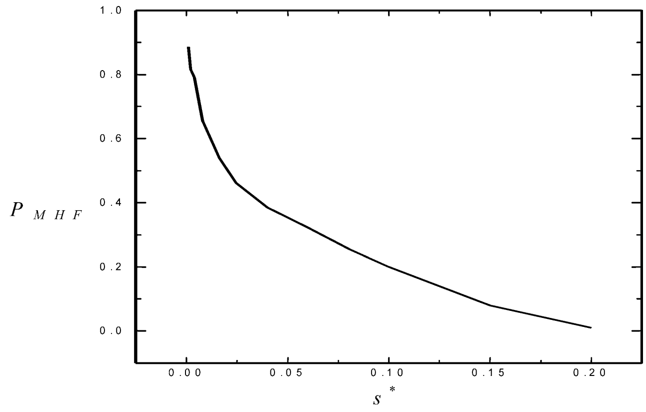

Fig. 5. Probability for transition to stable film boiling as a function of dry patch critical radius s^*.

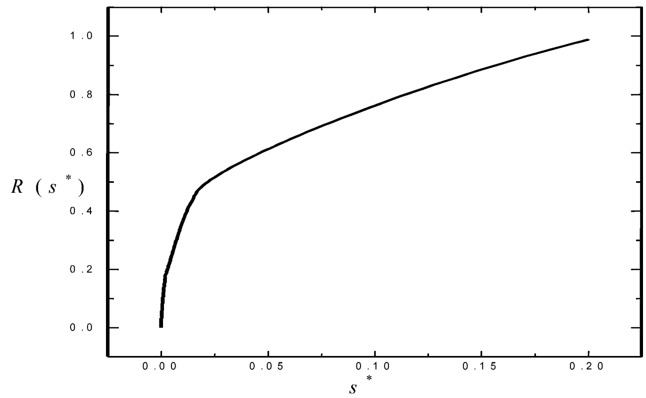

Fig. 6. Safety probability as a function of dry patch critical radius s^*.

5. Academics implications on classical MHF model

Transition to stable film boiling is analyzed from statistical viewpoint above, which challenges deterministic viewpoints in available transition boiling theory. For example, the MHF (minimal heat flux) model was traditionally viewed as a single point in the boiling plane (q and ΔT) and an independent entity [19]. In spite of its wide recognition and good agreement with experimental results, there are some reasons accounting for its invalidity. Because of the various parameters involved in the process and the potential dry patches interactions, it is useful to take a fresh look at the process. According to foregoing stochastic theory on transition boiling of this paper, the occurrence of MHF shows stochastic behaviors and is related to certain probability value. Consequently, MHF should be viewed as a region rather than a single point in the boiling plane (q and ΔT) and an independent entity. Restate, the occurrence of MHF should be considered within the region of the transition boiling. It is reasonable to describe the MHF phenomena as the high superheat transition boiling rather than as an independent outcome.

In addition, classical MHF model is based on the idea that MHF is purely the consequence of liquid–vapor interface destabilization, which mainly deals with liquid phase, vapor phase and their interface. Therefore, potential interactions with inclusion of heater are ignored. In fact, modeling MHF is a conjugate problem incorporating the liquid phase, vapor phase, solid phase and their interfaces [20]. In reality, the present model from stochastic view reflects the three phases and their interactions. It is natural that present stochastic analysis is more reasonable than classical MHF model. Stochastic model may lead to a new avenue of thought and research [21].

6. Comparisons to experimental results and potential for predicting transition boiling heat transfer

We previously conducted experimental investigations on transition and film boiling [22,23]. Here it is necessary to compare our present theoretical analysis to experimental results. We mainly mentioned two experimental results: (a) the scatters of data in our experiments obviously revealed the stochastic feature for the transition to stable film boiling [22], and we further found that the transition to film boiling was caused by un-rewet dry patches [23]. These results provided a strong experimental basis for our present theoretical analyses. (b) In our previous experiments [22,23], by adjusting factors such as velocity, sub-cooled degree, gravitational acceleration and other conditions, we had got different dry patches and critical dry patches features. Correspondingly, we had got different features for the transition to stable film boiling. Definite conclusions were drawn that transition boiling regime would be reduced with increasing flow velocity and sub-cooling degree and tend to disappear at extremely high flow velocity and sub-cooling degree [22]. The value of the critical dry patch quantitatively depends on some factors such as velocity, sub-cooled degree, gravitational acceleration et al, in our experiments. The increase of velocity, sub-cooled degree, gravitational acceleration and other conditions means that Eq. (23) have smaller values or Eq. (24) have larger values. Smaller values for Eq. (23) or larger values for Eq. (4) mean the reduction of transition boiling regime. Therefore, in this way our present theoretical analysis is in good agreement with experimental results in wider senses. We will conduct further experiments in the future. Of course, we hope our present theoretical analyses can be justified by other researchers' future experimental results, too. In fact, our previous theoretical model also justifies our present analysis [4].

Of course, the more important aspect is that present theory is potential for developing more accurate predictive models for transition boiling heat transfer. In the future, it is crucial to relate dry patch distribution model to heat transfer, though there still seems a long way to go.

7. Conclusions and recommendations

The development of transition boiling modes is virtually the formation and spreading process of dry patches under the action of superheat or heat fluxes. Different from available transition boiling analysis, when taking characteristics of nonlinear, nonequilibrium and stochastic into account, the occurrence of transition to stable film boiling is not deterministic but stochastic, which means the statistical analysis is not only beneficial to the current understanding of the transition boiling mechanisms, but also potential for the prediction of the transition boiling heat transfer with more accuracy in the future.

The ultimate goal of boiling heat transfer research is to be able to predict the boiling heat transfer rates without recourse to empiricism. Though much effort has been devoted toward achieving this goal, little effort has been made to understand the underlying nonlinear interactions and stochastic features. The present investigations in fact try to propose a new thinking way of developing transition boiling heat transfer research incorporating the nonlinear and stochastic effects. Further elaborate studies, such as photographic results are definitely required to form a new paradigm.

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